IRSTI 30.03.19

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https://doi.org/10.55956/HBEA9018

RESEARCH OF THE OPERATING MODE OF PERFORATORS

Abstract. The article presents the main provisions of the Partial Wave Method for solving direct and inverse problems for systems of hyperbolic equations. As an example, the problem of determining the geometry of a hammer by the shape of the first wave of the pulse generated by the hammer in a homogeneous cylindrical rod is considered.

Keywords: hyperbolic differential equations, longitudinal impact, graph-analytical methods of solution, inverse problems, waves.

Myasnikov A.A. Research of the operating mode of perforators //Mechanics and Technology / Scientific journal. – 2024. – No.4(86). – P.482-488. https://doi.org/10.55956/HBEA9018

Introduction. A wide class of problems in dynamics and technology is described by linear partial differential equations of hyperbolic type:

$$a_{11}(x,\tau)\frac{\partial^2 u(x,\tau)}{\partial x^2}+2a_{12}(x,\tau)\frac{\partial^2 u(x,\tau)}{\partial x\,\partial\tau}+a_{22}(x,\tau)\frac{\partial^2 u(x,\tau)}{\partial\tau^2}+$$
$$+b_1(x,\tau)\frac{\partial u(x,\tau)}{\partial x}+b_2(x,\tau)\frac{\partial u(x,\tau)}{\partial\tau}+c(x,\tau)u(x,\tau)+F(x,\tau)=0,$$
$$a_{12}^2-a_{11}a_{22}>0;$$

where: $u(x,\tau)$ – the desired function of two independent variables x and τ; $a_{11}(x,\tau)$, $a_{12}(x,\tau)$, $a_{22}(x,\tau)$, $b_1(x,\tau)$, $b_2(x,\tau)$, $c(x,\tau)$, $F(x,\tau)$ – known functions.

Euler showed that the solution of hyperbolic equations are wave-type objects. Apparently, there is no single strict definition of waves. It is preferable to be guided by the intuitive idea of a wave as a disturbance (or signal) moving along a medium with a certain determined speed. The disturbance can be of any type, provided that its localization is determined at any moment in time. The disturbance can be distorted, change its parameters, including the speed of propagation, but at the same time remain clearly distinguishable.

This approach may seem somewhat vague, but it is quite acceptable. Any attempt to give a more rigorous definition leads to significantly greater limitations.

Materials and methods. The concept of wave motion is one of the most complex scientific concepts. On the one hand, wave motion is often limited to a description with a set of more or less substantiated hypotheses. On the other hand, wave motion is studied by many disciplines, since wave processes are encountered in almost all areas of science and technology. Although the phenomena often have unique features, it was possible to develop general approaches to mathematical modeling of wave processes.

Waves described by hyperbolic partial differential equations are usually called hyperbolic waves.

Solutions to partial differential equations do not determine the desired functions in the full sense, but only determine some properties of the class of functions satisfying the equations. The functions themselves are further defined by initial and boundary conditions.

Application of the partial wave method to solve direct and inverse problems of systems of hyperbolic partial differential equations is considered in relation to problems of mathematical modeling of impact systems for technological purposes.

The general scheme of impact systems is shown in Figure 1.

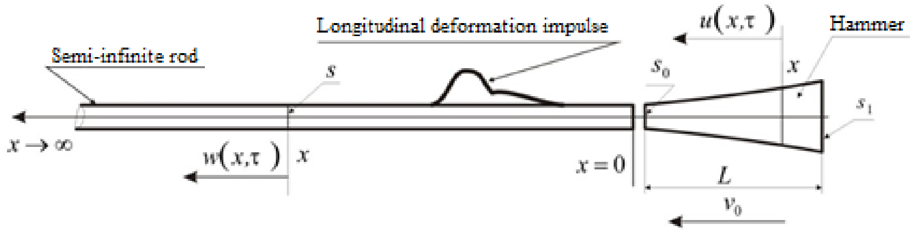


Fig. 1. Scheme of technological impact systems

A hammer, which is a short rod of variable cross-section – a hammer – strikes a long semi-infinite cylindrical rod. If the material of the rods is homogeneous, the ends are flat, then the dynamics of the interaction are described by the following system of equations, in the coordinate system shown in Figure 1:

$$\frac{\partial^2 w(x,t)}{\partial x^2}+\frac{d\,s(x)}{d\,x}\frac{1}{s(x)}\frac{\partial w(x,t)}{\partial x}-\frac{\partial^2 w(x,t)}{\partial t^2}=0\ , \quad \frac{\partial^2 u(x,t)}{\partial x^2}-\frac{\partial^2 u(x,t)}{\partial t^2}=0$$

with initial conditions:

$$u(x,0)=0, \quad w(x,0)=0, \quad \frac{\partial u(x,0)}{\partial t}=0, \quad \frac{\partial w(x,0)}{\partial t}=\frac{v_0}{c};$$

and boundary conditions:

$$\frac{\partial w(-L,t)}{\partial x}=0\ , \quad u(0,t)=w(0,t)\ , \quad s\,\frac{\partial u(0,t)}{\partial x}=s_0\,\frac{\partial w(0,t)}{\partial x},$$

$$\lim_{x\to\infty}\frac{\partial u(x,t)}{\partial x}=0\,;$$

where: $w(x,\tau)$ – function of displacement of the hammer cross-section with coordinate x at a point in time τ; $u(x,\tau)$ – rod cross-section displacement function; $s(x)$ – hammer cross-sectional area function; s – cross-sectional area of a semi-infinite rod; $t=c\tau$; $c=\sqrt{\frac{E}{\rho}}$ – the velocity of propagation of longitudinal vibrations in a rod with a modulus of elasticity *E*, density ρ; s_0 – hammer striking face area.

The direct problem is the determination of the relative deformation function in a semi-infinite rod. $\varepsilon_u(x,t)=\frac{\partial u(x,t)}{\partial x}$ developed by the impulse of dynamic deformation generated by the longitudinal impact of a hammer of a given geometry.

The inverse problem is the determination of the geometry of a hammer through the function of the cross-sectional area of the hammer $s(x)$ according to the known function of relative deformation in a semi-infinite rod $\varepsilon_u(x,t)=\frac{\partial u(x,t)}{\partial x}$ developed by the impulse of dynamic deformation.

The method of partial waves is based on the position that longitudinal dynamic deformation of homogeneous rods is described by the equation:

$$\frac{\partial^2 u(x,t)}{\partial x^2}-\frac{\partial^2 u(x,t)}{\partial t^2}=0,$$

the solution of which can be represented as:

$$u(x,\tau)=f_+(x-t)+f_-(x+t);$$

where function $f_+(x-t)$ defines a wave moving along a rod without distortion in the positive direction of the coordinate axis of the cross sections x; $f_-(x+t)$ – a wave moving in the negative direction of the coordinate axis, as shown by L. Euler.

The process of generating longitudinal waves by impact is determined from the condition of continuity of the flow of forces and displacements at the impact ends of the rods, (Fig. 2).

If the colliding rods n and $n+1$ or the impact stages are cylindrical, then the relative deformations developed by the waves will be constant and can be represented by the formulas:

$$\varepsilon_{n+1}=\varepsilon_+\cdot\overline{\eta}(t-x),$$
$$\varepsilon_n=\varepsilon_-\cdot\overline{\eta}(t+x),$$

where: $\overline{\eta}(z)=\begin{cases}1, & npu\ z>0\\ 0, & npu\ z<0\end{cases},$

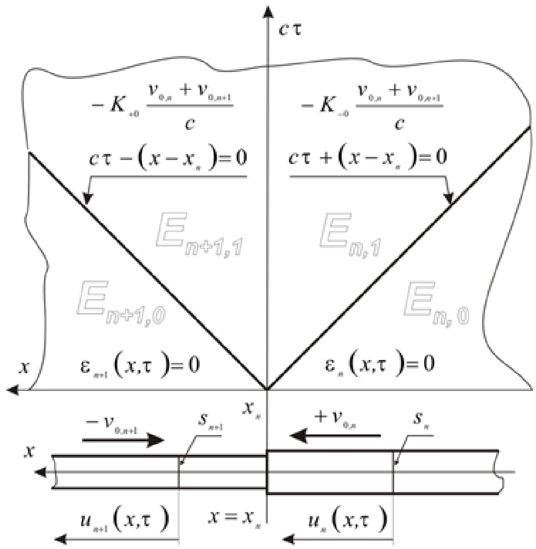


Fig. 2. To the analysis of the generation of dynamic deformation waves of cylindrical rods under longitudinal impact

Heaviside impulse function.

From the equations defining the continuity of the flows of forces and displacements, the functions of the waves generated at the junction of the rod steps are found from the moment a certain generating wave reaches the boundary of the steps. The analysis shows that the waves generated at the junction of cylindrical steps develop relative deformations of constant magnitude.

Thus, an accurate analysis is constructed in the case of a collision of stepped cylindrical rods, naturally, within the accuracy of the longitudinal dynamic modeling model of the rods.

It can be assumed that in the model, the replacement of rods with curvilinear generatrices of the lateral surface by stepped cylindrical rods can be quite correct. In this case, the main system will be replaced by an infinitely finite system of equations:

$$\frac{\partial^2 u_1(x,t)}{\partial x^2} - \frac{\partial^2 u_1(x,t)}{\partial t^2} = 0,$$

$$\frac{\partial^2 u_2(x,t)}{\partial x^2} - \frac{\partial^2 u_2(x,t)}{\partial t^2} = 0,$$

$$\dots$$

$$\frac{\partial^2 u_m(x,t)}{\partial x^2} - \frac{\partial^2 u_m(x,t)}{\partial t^2} = 0;$$

with a correspondingly modified system of boundary conditions.

Research results and discussion. It is quite difficult to trace analytically and even more so to rationally present the solution of the direct problem. Graphical presentation of part of the information significantly simplifies the presentation of the solution. The coordinate plane is divided by characteristics corresponding to the Heaviside functions into triangular regions, where the values corresponding to the relative deformations developed by partial waves have the same value. An example of the solution of the direct problem is shown in Figure 3.

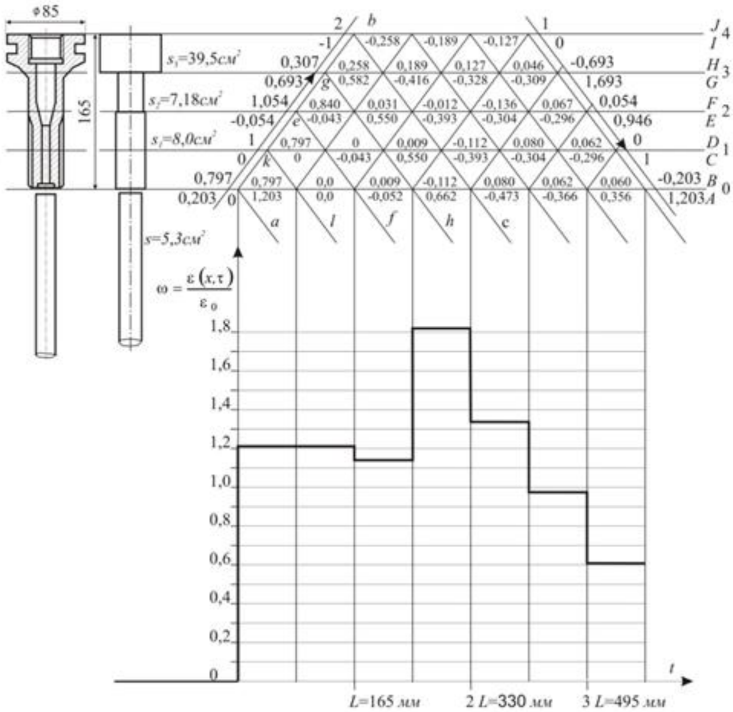


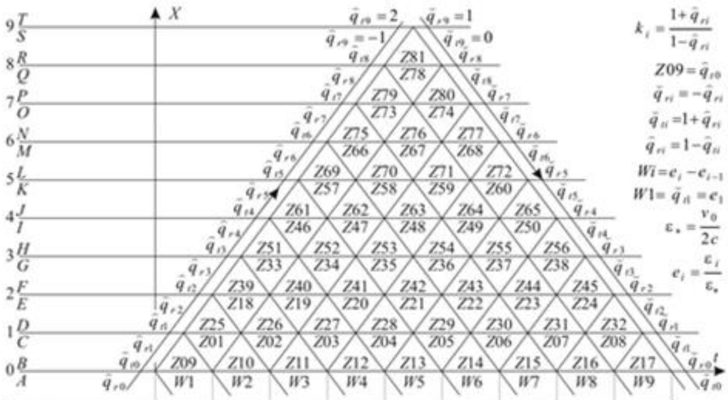
Fig. 3. Graph-analytical solution of the direct problem by the partial wave method

When constructing an algorithm for solving direct problems using the partial wave method, dependencies were discovered that made it possible to create an algorithm for solving inverse problems.

An example of solving an inverse problem is shown in Figure 4. In this case, this is a solution to the problem of determining the geometry of a hammer generating a “triangular” pulse, i.e. a pulse with a linearly increasing value of the relative deformation developed by the first wave.

The relative deformation function was chosen as the desired one. Modern experimental equipment records exactly the relative deformations on the surface of the rods. The remaining functions characterizing the pulse of longitudinal dynamic deformation: stress, velocity, effort are linearly related to the function of relative deformation.

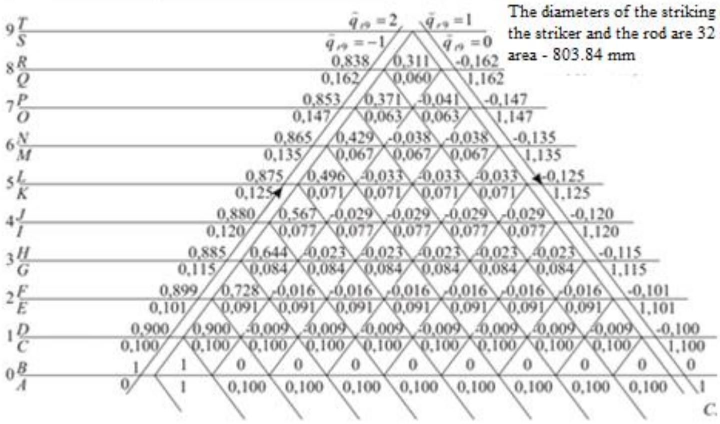
Conclusion. To check the accuracy of the method, analytical solutions can be used to determine the pulses of dynamic deformation generated in a semi-infinite rod by a longitudinal blow of hammers with curvilinear generators of the lateral surface.



A.

Diameters of circular sections, mm	32	35	39	44	50	57	65	75	104
Areas of circular cross-sections, mm	803,84	980,68	1201,33	1513,68	1926,91	2478,01	3251,15	4372,80	6965,07
Area ratio k_i	1,220	1,225	1,260	1,273	1,286	1,312	1,345	1,387	0
Orthation coefficient $\bar{q}_{ri}$	0,100	0,101	0,115	0,120	0,125	0,135	0,147	0,162	-1,000
Wave state in the barbell Wi	$W1=1$	$W2=0,1$	$W3=0,1$	$W4=0,1$	$W5=0,1$	$W6=0,1$	$W7=0,1$	$W8=0,1$	$W9=0,1$
Form function e_i	$e_1=1$	$e_2=1,1$	$e_3=1,2$	$e_4=1,3$	$e_5=1,4$	$e_6=1,5$	$e_7=1,6$	$e_8=1,7$	$e_9=1,8$
Deformation ε_i									

B.



C.

Fig.4. Solution of the inverse problem of longitudinal collision of rods by the partial wave method

References

1. Agafonov S.A., German A.D Muratova T.V. Differentsial'nyye uravneniya [Differential Equations]: textbook for universities. 2nd ed. /Ed. by V.S. Zarubin, A.P. Krishchenko. – Moscow: Publishing House of Bauman Moscow State Technical University, 2000. – 348 p. [in Russian].
2. Tereshchenko S.V. Obyknovennyye differentsial'nyye uravneniya pervogo po-ryadka [Ordinary Differential Equations of the First Order]: textbook-method manual for solving problems. – Apatity, Publishing House of the KF PetrSU, 2003. – 75 p. [in Russian].

Material received on 20.12.24.

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ПЕРФОРАТОРДЫҢ ЖҰМЫС РЕЖИМІН ЗЕРТТЕУ

Аңдатпа. Гиперболалық типті теңдеулер жүйесінің тура және қайтарымды есептерін шешуге арналған жартылай толқындар әдісінің негізгі принциптері келтірілген. Мысал ретін-де біртекті цилиндрлік өзекшедегі балғамен тудыратын импульстің бірінші толқыны түріндегі балғаның геометриясын анықтау мәселесі қарастырылған.

Тірек сөздер: гиперболалық типті дифференциалдық теңдеулер, бойлық әсер ету, шешудің графи-калық-аналитикалық әдістері, кері есептер, толқындар.

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ИССЛЕДОВАНИЯ РЕЖИМА РАБОТЫ ПЕРФОРАТОРОВ

Аннотация. Изложены основные положения метода парциальных волн для решения прямых и обратных задач систем уравнений гиперболического типа. В качестве примера рас-смотрена задача определения геометрии молотка по форме первой волны импульса, генерируемого молотком в однородном цилиндрическом стержне.

Ключевые слова: дифференциальные уравнения гиперболического типа, продольный удар, графоана-литические методы решения, обратные задачи, волны.