IRSTI 30.15.27

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https://doi.org/10.55956/TXFX9931

ON MODELS OF LONGITUDINAL VIBRATIONS OF INHOMOGENEOUS RODS

Abstract. The main provisions of the model of longitudinal vibrations of rods are presented, taking into ac-count certain phenomena. To construct a model of longitudinal vibrations of rods from the system, the method of successive approximation was used. Models are based on three components: the basic law or principle of dynamics, a hypothesis or basic statement, and a construction algorithm method.

Keywords: model, rod, longitudinal vibrations, geometric parameters, Newton's law, stress, material density.

Myasnikov A.A. On models of longitudinal vibrations of inhomogeneous rods // Mechanics and Technology / Scientific journal. – 2024. – No.1(83). – P.292-300.
https://doi.org/10.55956/TXFX9931

Introduction. Models of longitudinal elastic vibrations of rods are used for calculations in various fields of science and technology. The geometric parameters of the objects under study vary over a wide range - from trains to nanotubes. When constructing models of longitudinal vibrations of rods taking into account certain phenomena, it is useful to take into account the logic of their construction. Models are based on three components: the basic law or principle of dynamics, a hypothesis or basic statement, and a construction algorithm method.

Most of the models of longitudinal vibrations of rods are variations of the model of longitudinal vibrations of an ideal one-dimensional line. Newton's second law is used as the main one, written for an element of a material line of infinitesimal length cut out by two cross sections, with coordinates and mass, Figure 1.

Research conditions and methods. The resultant forces are determined by the forces equivalent to the action of the rejected parts along the axis of the rod. In order to maintain the general logic of presentation, it is assumed that the material line has an infinitesimal cross-sectional area. Mass of the cut element where is the volumetric density of the line material. Let the displacement of a point of a material line with coordinate at the moment of time be, then the acceleration of the selected element will be determined by the formula. The given relations determine the equation.

Most of the models of longitudinal vibrations of rods are variations of the model of longitudinal vibrations of an ideal one-dimensional line. Newton's second, $F = d\,m\,a$, law is used as the main one, written for an element of a material line of infinitesimal length cut out by two cross sections $d\,x$, with coordinates $x + d\,x$ and x, mass $d\,m$, Figure 1.

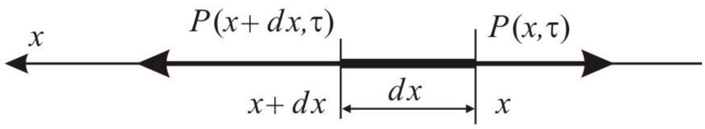

Fig. 1. Calculation diagram for the model of longitudinal vibrations of a one-dimensional material line

Conditions and methods of research. v Resultant force F determined by forces equivalent to the action of the rejected parts $F = P\left(x + d\,x\right) - P\left(x\right)$ along the axis of the rod x. In order to preserve the general logic of presentation, it is assumed that the material line has an infinitesimal cross-sectional area $\Delta\,s$. Weight of cut element $d\,m = \rho\,\Delta s\,d\,x$ where ρ - bulk density of line material. Let $u\left(x,\tau\right)$ displacement of a material line point with coordinate x at a point in time τ, then the acceleration of the selected element is determined by the formula $a = \dfrac{\partial^2 u\left(x,\tau\right)}{\partial \tau^2}$. The above relations determine the equation

$$\frac{P\left(x + d\,x,\tau\right) - P\left(x,\tau\right)}{d\,x} = \rho\,\Delta\,s\,\frac{\partial^2 u\left(x,\tau\right)}{\partial \tau^2} \tag{1}$$

or $\dfrac{\partial P\left(x,\tau\right)}{\Delta s\,\partial x} = \rho\,\dfrac{\partial^2 u\left(x,\tau\right)}{\partial \tau^2}$. Injection voltage $\sigma\left(x,\tau\right) = \dfrac{P\left(x,\tau\right)}{\Delta s}$, brings the equation to the form:

$$\frac{\partial \sigma\left(x,\tau\right)}{\partial x} = \rho\,\frac{\partial^2 u\left(x,\tau\right)}{\partial \tau^2} \tag{2}$$

If the stresses are caused by linear elastic deformations, then, based on Hooke's law, the relation holds: $\sigma\left(x,\tau\right) = E\,\varepsilon\left(x,\tau\right)$ where E - modulus of elasticity of the first kind of material line, $\varepsilon\left(x,\tau\right) = \dfrac{\partial u\left(x,\tau\right)}{\partial x}$ - longitudinal relative deformation.

As a result, equation (1) will take the classical form:

$$\frac{\partial}{\partial x}\left(E\,\frac{\partial u\left(x,\tau\right)}{\partial x}\right) = \rho\,\frac{\partial^2 u\left(x,\tau\right)}{\partial \tau^2} \tag{3}$$

If $E = const$, $\rho = const$ then the equation can be rewritten as:

$$\frac{\partial^2 u(x,\tau)}{\partial x^2} = \frac{\partial^2 u(x,\tau)}{c^2\,\partial \tau^2} \text{ where } c = \sqrt{\frac{E}{\rho}} \qquad (4)$$

Euler's analysis of equation (4) shows:

- the solution to the equation can be presented in the form: $u(x,\tau) = f_+(x - c\tau) + f_-(x + c\tau)$

where $f_+(x - c\tau)$, $f_-(x + c\tau)$ - indeterminate functions of displacements of points of a material line under the influence of longitudinal oscillations moving in the positive and negative directions of the coordinate axis, respectively;

- c – velocity of propagation of longitudinal vibrations;

- longitudinal vibrations move in a homogeneous material line without distortion of shape and attenuation.

In order to generalize the equation to the case of longitudinal vibrations of inhomogeneous rods, the hypothesis of flat sections was introduced: the sections of the rods remain flat during longitudinal vibrations. The consequences of the hypothesis can be formulated as follows:

- all points of the cross section of a straight rod during longitudinal vibrations have the same displacements, velocities, accelerations;

- in the vicinity of the section the stresses and relative strains are equal.

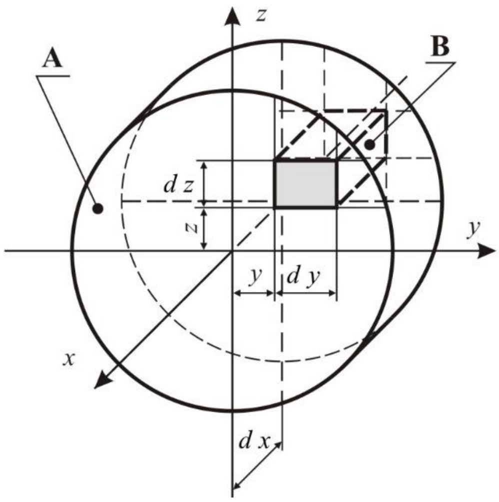

Fig. 2. Calculation scheme for models of longitudinal dynamic deformation of rods

Equation (2) in this case will determine the dynamics of element B of the rod in the direction of the coordinate axis of the cross sections x.

If we assume that the rod has a variable cross-sectional area $s(x)$, then equation (1) will be rewritten as: $\frac{\partial P(x,\tau)}{\partial x} = \rho\, s(x) \frac{\partial^2 u(x,\tau)}{\partial \tau^2}$ or

$$\frac{\partial}{\partial x}\left(E\, s(x) \frac{\partial u(x,\tau)}{\partial x} \right) = \rho\, s(x) \frac{\partial^2 u(x,\tau)}{\partial \tau^2} \qquad (5)$$

If $E = const$, $\rho = const$ then the equation will be rewritten as

$$\frac{\partial^2 u(x,\tau)}{\partial x^2}+\frac{d\ln(s(x))}{dx}\frac{\partial u(x,\tau)}{\partial x}-\frac{\partial^2 u(x,\tau)}{\partial t^2}=0 \qquad (6)$$

A model of longitudinal vibrations of a rod can be constructed on the basis of the theorem on the change in momentum for a continuous medium, while maintaining the hypothesis of plane sections and the method of sections. To do this, select a rod element limited by the lateral surface and arbitrary flat cross sections with coordinates x_1 and x_2, $x_2 > x_1$ length $l = x_2 - x_1$. In this case, the equation for longitudinal vibrations of a rod of variable cross-section is obtained in integral-differential (balance) form:

$$E\,s(x_2)\frac{\partial u(x_2,\tau)}{\partial x}-E\,s(x_1)\frac{\partial u(x_1,\tau)}{\partial x}=\int_{x_1}^{x_2}\rho\,s(x)\frac{\partial^2 u(x,\tau)}{\partial \tau^2}dx \qquad (7)$$

If the upper limit is considered variable $x_2 = x$, then differentiation by the upper limit of both parts (7) leads to the model equation according to the hypothesis of plane sections (5).

The equation in the form (7) can give more accurate results when solving problems using numerical methods, when the continuous domain of definition of the displacement function $u(x,\tau)$ replaced by a discrete mesh. For example, when solving by the finite difference method.

The above models do not take into account the experimentally established law connecting linear relative deformations in the longitudinal and transverse directions:

$$\varepsilon_r = -\mu\,\varepsilon_x \qquad (8)$$

where μ - Poisson's ratio; ε_x - relative deformation in axial direction; ε_r - relative deformation in the radial direction.

One of the options for taking this law into account is the introduction of the Rayleigh correction, which takes into account the kinetic energy of the radial motion of the particles of the rod. While maintaining the hypothesis of plane sections and the section method, the model with the Rayleigh correction is usually built on the Hamilton-Ostrogradsky variational principle: The actual motion of a system with holonomic connections differs from other kinematically possible movements in that for it the variations of the action according to Hamilton-Ostrogradsky, specific for an arbitrary period of time $W = \int_{\tau_0}^{\tau} L\,d\tau$ *equal to zero* $\delta\int_{\tau_0}^{\tau} L\,d\tau = 0$, $L = T - \Pi$ - Lagrange function, determined by the difference between the kinetic N and potential Π energies of the system.

The Lagrange function is determined for a rod element limited by two cross sections of length L. The result is:

$$\delta\int_{\tau_0}^{\tau}\int_{(l)}\frac{1}{2}\left[\rho\, s(x)\left(\frac{\partial u(x,\tau)}{\partial\tau}\right)^2+\rho\,\mu^2 J_\rho(x)\left(\frac{\partial^2 u(x,\tau)}{\partial x\,\partial\tau}\right)^2-\right.$$
$$\left.-E\,s(x)\left(\frac{\partial u(x,\tau)}{\partial x}\right)^2\right]dx\,d\tau=0$$

which is equivalent to the condition for the extremum of the functional:

$$-\frac{\partial}{\partial x}\left\{\frac{\partial L}{\partial\varepsilon_x}\right\}-\frac{\partial}{\partial\tau}\left\{\frac{\partial L}{\partial v}\right\}+\frac{\partial^2}{\partial x\,\partial\tau}\left\{\frac{\partial L}{\partial v_r}\right\}=0$$

Accordingly, the equation of longitudinal dynamic deformation of a rod of variable cross-section takes the form:

$$\frac{\partial}{\partial x}\left(E\,s(x)\frac{\partial u(x,\tau)}{\partial x}\right)-\rho\,s(x)\frac{\partial^2 u(x,\tau)}{\partial\tau^2}+$$
$$+\rho\,\mu^2\frac{\partial}{\partial x}\left(J_\rho(p)\frac{\partial^3 u(x,\tau)}{\partial x\,\partial\tau^2}\right)=0 \qquad (9)$$

The Rayleigh correction, in relation to the equation of longitudinal vibrations according to the hypothesis of plane sections (3), adds a term linearly related to the fourth-order derivative

$$+\rho\,\mu^2\frac{\partial}{\partial x}\left(J_\rho(p)\frac{\partial^3 u(x,\tau)}{\partial x\,\partial\tau^2}\right)$$

This correction takes into account, to some extent, the geometry of the cross section of the rod through the function of the polar moment of inertia $J_\rho(p)$. The equation can be simplified if the mechanical properties of the material E and ρ do not depend on the section coordinates x and time τ. At zero Poisson's ratio, the Rayleigh correction is reduced to zero.

Relatively few solutions to problems using this model are known due to its complexity.

The mechanics of a solid deformed body determines the dynamics of element B in the Cartesian coordinate system in displacements by the following system of equations:

$$\frac{\partial u_x}{\partial x}=\frac{1}{E}\left[\sigma_x-\mu\left(\sigma_y+\sigma_z\right)\right] \qquad (10)$$

$$\frac{\partial u_y}{\partial y}=\frac{1}{E}\left[\sigma_y-\mu\left(\sigma_z+\sigma_x\right)\right] \qquad \frac{\partial u_z}{\partial z}=\frac{1}{E}\left[\sigma_z-\mu\left(\sigma_x+\sigma_y\right)\right] \qquad (11)$$

$$\frac{\tau_{xy}}{G}=\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial y} \qquad \frac{\tau_{yz}}{G}=\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z} \qquad \frac{\tau_{zx}}{G}=\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x} \qquad (12)$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho \frac{\partial^2 u_x}{\partial \tau^2} \quad (13)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \rho \frac{\partial^2 u_y}{\partial \tau^2} \qquad \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = \rho \frac{\partial^2 u_z}{\partial \tau^2} \quad (14)$$

where σ_k - stress in the direction normal k to the section; τ_{jk} - shear stress in direction j in section with the normal k ; u_k - point displacement in direction k ; G - shear modulus, or second modulus of elasticity.

The equation (12) $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho \frac{\partial^2 u_x}{\partial \tau^2}$ determines the dynamics of element B as well as the equation (2) $\frac{\partial \sigma(x,\tau)}{\partial x} = \rho \frac{\partial^2 u(x,\tau)}{\partial \tau^2}$ according to the hypothesis of plane cross sections.

That is, models based on the hypothesis of flat sections do not take into account at least tangential stresses.

Research results. To construct a model of longitudinal vibrations of rods from system (10)-(14), we apply the method of successive approximation. The basic statement is that the components of the radial stress in the zero approximation are equal to zero: $\sigma_y = 0$, $\sigma_z = 0$. With static longitudinal deformation of the rod, this statement is certainly true, as follows from the full formulation of Hooke's law. In the case of dynamic longitudinal deformation, the statement can only be accepted as a zero approximation. The basic statement is obviously only approximately true, so calling the basic statement a hypothesis is apparently incorrect due to its obviously approximate nature.

To construct a rod model, equation (12) for element B is integrated over the volume of element A of the rod:

$$\frac{\partial}{\partial x} \int\limits_{(s(x))} \sigma_x \, d s_x + \int\limits_{(s(x))} \frac{\partial \tau_{xy}}{\partial y} d s_x + \int\limits_{(s(x))} \frac{\partial \tau_{zx}}{\partial z} d s_x - \int\limits_{(s(x))} \rho \frac{\partial^2 u_x}{\partial \tau^2} d s_x = 0 \quad (15)$$

where $d s_x$ - area of the element face B perpendicular to the coordinate axis of the cross sections x .

$$\frac{\partial}{\partial x}\left(E \frac{\partial u_x}{\partial x} \int\limits_{(s(x))} d s_x + \frac{\partial}{\partial x}\left[\frac{\mu^2}{2}\left(G \frac{\partial^3 u_x}{\partial x^3} - \rho \frac{\partial^3 u_x}{\partial x \partial \tau^2} \right) \int\limits_{(s(x))} [(y^2 + z^2) - \right.\right.$$
$$\left. - (y_*^2 + z_*^2)] d s_x \right] + \frac{\partial}{\partial y}\left(-\mu G \frac{\partial^2 u_x}{\partial x^2} y \right) \int\limits_{(s(x))} d s_x +$$
$$+ \frac{\partial}{\partial z}\left(-\mu G \frac{\partial^2 u_x}{\partial x^2} z \right) \int\limits_{(s(x))} d s_x - \rho \frac{\partial^2 u_x}{\partial \tau^2} \int\limits_{(s(x))} d s_x = 0 \quad (16)$$

Integral $\int\limits_{(s(x))} d s_x = s(x)$ represents the cross-sectional area. To transform the integral $J_{d1}(x) = \int\limits_{(s(x))} \left[\left(y^2 + z^2 \right) - \left(y_*^2 + z_*^2 \right) \right] d s_x$ distance is entered r from the coordinate axis x to the point with coordinates y, z belonging to the cross section with coordinate x: $r^2 = y^2 + z^2$ and r_* - distance from axis x to a fixed point on the outer contour of the section, $r_*^2 = y_*^2 + z_*^2$. If the section is a circle, then r_* determines the radius of the circle. As a result, the integral is transformed to the form $J_{d1}(x) = \int\limits_{(s(x))} r^2 d s_x - r_*^2 \int\limits_{(s(x))} d s_x$ Integral $J_\rho(x) = \int\limits_{(s(x))} r^2 d s_x$ determines the polar moment of inertia of the section, therefore $J_{d1}(x) = J_\rho(x) - r_*^2 s(x)$. For a circular cross section, the polar moment of inertia is given by: $J_{\rho\,кр} = \pi r_*^4 / 2$ hence:

$$J_{d1кр}(x) = \pi \frac{r_*^4}{2} - r_*^2 \pi r_*^2 = -\pi \frac{r_*^4}{2} \;\Rightarrow\; J_{d1кр}(x) = -J_{\rho\,кр}(x)$$

Thus, the equation for longitudinal vibrations of the deformation of a rod of variable cross-section according to this model, to a first approximation, takes the form:

$$\frac{\partial}{\partial x}\left(E\, s(x) \frac{\partial u_x}{\partial x} \right) - \frac{\partial}{\partial x}\left[\frac{\mu^2}{2} J_{d1} \left(G \frac{\partial^3 u_x}{\partial x^3} - \rho \frac{\partial^3 u_x}{\partial x\, \partial \tau^2} \right) \right] - \\ - 2 \mu G\, s(x) \frac{\partial^2 u_x}{\partial x^2} - \rho\, s(x) \frac{\partial^2 u_x}{\partial \tau^2} = 0 \qquad (17)$$

The equation includes both the Rayleigh correction and a number of others relative to the model based on the hypothesis of plane sections.

If we use the obtained formulas as initial ones and repeat the derivation algorithm, we can obtain the equation of longitudinal vibrations in the second approximation:

$$\frac{\partial}{\partial x}\left(E\, s(x) \frac{\partial u_x}{\partial x} \right) - \frac{\partial}{\partial x}\left[\frac{\mu^2 (1-\mu)}{48} J_{d21}(x) \left(G \frac{\partial^5 u_x}{\partial x^5} - \right.\right. \\ \left.\left. - 2\rho \frac{\partial^5 u_x}{\partial x^3 \partial \tau^2} + \frac{\rho^2}{G} \frac{\partial^5 u_x}{\partial x\, \partial \tau^4} \right) \right] + \frac{\partial}{\partial x}\left[\frac{\mu^3}{4} J_{d22}(x) \left(G \frac{\partial^5 u_x}{\partial x^5} - 2\rho \frac{\partial^5 u_x}{\partial x^3 \partial \tau^2} + \right.\right. \\ \left.\left. + \frac{\rho^2}{G} \frac{\partial^5 u_x}{\partial x\, \partial \tau^4} \right) \right] + \frac{\partial}{\partial x}\left[\frac{\mu^2 (1+\mu)}{2} J_{d1}(x) \left(G \frac{\partial^3 u_x}{\partial x^3} - \rho \frac{\partial^3 u_x}{\partial x\, \partial \tau^2} \right) \right] + \\ + \frac{\mu (1 - 2\mu)}{4} J_{d1}(x) \left(G \frac{\partial^4 u_x}{\partial x^4} - \rho \frac{\partial^4 u_x}{\partial x^2 \partial \tau^2} \right) - 2 \mu G\, s(x) \frac{\partial^2 u_x}{\partial x^2} - \\ - \rho\, s(x) \frac{\partial^2 u_x}{\partial \tau^2} = 0 \qquad (18)$$

where $J_{d21}(x)=\int\limits_{(s(x))}\left[\left(y^4+z^4\right)-6\left(y_*^2\,y^2+z_*^2\,z^2\right)+5\left(y_*^4+z_*^4\right)\right]d\,s_x$

$J_{d22}(x)=\int\limits_{(s(x))}\left(y^2-y_*^2\right)\left(z^2-z_*^2\right)d\,s_x$ - geometric characteristics of flat sections.

Discussion of scientific results. A comparison of equations (17) and (18) shows that at the second step of approximation, all terms of the first approximation are retained and terms containing derivatives of a higher order are added, which determines the convergence of the model algorithm using the method of successive approximation to arbitrarily accurate, in the limit, Eq.

To solve technical problems and compare all the above models, it is enough to keep the terms including derivatives no higher than the second order:

$$\frac{\partial}{\partial x}\left(E\,s(x)\frac{\partial u_x}{\partial x}\right)-2\,\mu\,G\,s(x)\frac{\partial^2 u_x}{\partial x^2}-\rho\,s(x)\frac{\partial^2 u_x}{\partial \tau^2}=0 \tag{19}$$

When replacing u_x on $u(x,\tau)$, taking into account the ratio $G=E/\left[2\left(1+\mu\right)\right]$, the equation will be rewritten as:

$$\frac{E}{1+\mu}\,s(x)\frac{\partial^2 u(x,\tau)}{\partial x^2}+E\,\frac{d\,s(x)}{d\,x}\frac{\partial u(x,\tau)}{\partial x}-\rho\,s(x)\frac{\partial^2 u(x,\tau)}{\partial \tau^2}=0 \tag{20}$$

If the rod is geometrically homogeneous, i.e. the cross section does not change along the length, $s(x)=const$, then the equation is transformed to the form:

$$\frac{E}{1+\mu}\frac{\partial^2 u(x,\tau)}{\partial x^2}-\rho\,\frac{\partial^2 u(x,\tau)}{\partial \tau^2}=0 \tag{21}$$

coinciding, in structure, with the equation of longitudinal vibrations of a material line. The model determines the speed of propagation of longitudinal vibrations using a slightly different formula: $c_1=\sqrt{E/\left[\left(1+\mu\right)\rho\right]}$, than according to the hypothesis of plane sections $c=\sqrt{E/\,\rho}$. For materials with zero Poisson's ratio, $\mu=0$, the equations for all models are the same.

When introducing a variable $t=c_1\,\tau$, which can be interpreted as the distance over which the dynamic deformation moves over time τ, equation (20) will be rewritten as:

$$s(x)\frac{\partial^2 u(x,t)}{\partial x^2}+\left(1+\mu\right)\frac{d\,s(x)}{d\,x}\frac{\partial u(x,t)}{\partial x}-s(x)\frac{\partial^2 u(x,t)}{\partial t^2}=0,$$

or, in a more compact form:

$$\frac{\partial^2 u(x,t)}{\partial x^2}+\left(1+\mu\right)\frac{d\,ln\left(s(x)\right)}{d\,x}\frac{\partial u(x,t)}{\partial x}-\frac{\partial^2 u(x,t)}{\partial t^2}=0 \tag{22}$$

Conclusion. For definiteness, equation (22) is proposed to be called ORN (no radial stress) the equation of longitudinal vibrations of geometrically non-uniform rods made of homogeneous material under linear elastic deformations. In English: RSA (radial stresses are absent) equation of longitudinal vibration of rods (bars).

In the case of inhomogeneities of various kinds, or a nonlinear dependence of stresses on deformations, the algorithm for constructing the model is preserved with appropriate adjustment of the initial equations of the system.

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Material received on 29.03.24.

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ГЕТЕРОГЕНДІ ӨЗЕКТЕРДІҢ БОЙЛЫҚ ТЕРБЕЛІСТЕРІНІҢ МОДЕЛЬДЕРІ ТУРАЛЫ

Аңдатпа. Белгілі бір құбылыстарды ескере отырып, шыбықтардың бойлық тербелісі моделінің негізгі ережелері көрсетілген. Жүйеден өзектердің бойлық тербелістерінің моделін құру үшін дәйекті жуықтау әдісі қолданылды. Модельдер үш құрамдас бөлікке негізделген: негізгі заң немесе динамика принципі, гипотеза немесе негізгі мәлімдеме, құрылыс алгоритмі әдісі.

Тірек сөздер: модель, өзек, бойлық тербелістер, геометриялық параметрлер, Ньютон заңы, кернеулер, материалдың тығыздығы.

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О МОДЕЛЯХ ПРОДОЛЬНЫХ КОЛЕБАНИЙ НЕОДНОРОДНЫХ СТЕРЖНЕЙ

Аннотация. Изложены основные положения модели продольных колебаний стержней с учетом тех или иных явлений. Для построения модели продольных колебаний стержней из системы применен метод последовательного приближения. Модели основываются на трех составляющих: основной закон или принцип динамики, гипотеза или базовое утверждение, метод алгоритма построения.

Ключевые слова: модель, стержень, продольные колебания, геометрические параметры, закон Ньютона, напряжения, плотность материала.