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## **A REVIEW ON RESEARCH OF STUDENT EVALUATION OF MATHEMATICAL EXPLANATIONS IN MATHEMATICS CLASSROOM**

**Abstract.** This article reviews the previous publications in the area of evaluation of mathematical explanations by students in an Inquiry-Based Mathematics Classroom. Many students are hesitant to explain their work and question their classmates' explanations. Besides students rarely explain a problem solution and rarely evaluate mathematical explanations. Along with the students' background, educational and socio-cultural factors, these factors - sociomathematical norms, teaching authority and classroom mathematical practices - also affect the academic performance of students.

The aim of the paper is to identify the scientific productivity and impacts of researchers in the field of student evaluation of mathematical explanations in an Inquiry-Based Mathematics Classroom in the last 20 years (2002-2022). In this work, the results and analysis of a review is reported. The application of the limited type of bibliometric method is best suited for answering the research questions.

The Google Scholar database was used to search for publications on the subject of student evaluation of mathematical explanations.

The results of the present work show that the largest number of research papers concerns sociomathematical norms in the math classroom and the negative impact of these norms in the process of student evaluation of mathematical explanations, and there are relatively few reports of applying the qualitative analysis (case studies) in research on student evaluation of math explanations.

**Keywords:** evaluation of mathematical explanations; Sociomathematical norms; Teaching authority; Classroom mathematical practices; Inquiry-based mathematics classroom.

**Introduction.** The National Council of Teachers of Mathematics (NCTM) is the world's largest mathematics education organization. According to the document, published by this organization in 2000 (Standards for School Mathematics), "the ability to reason is an essential part of understanding mathematics". Doing mathematics is not just about performing procedures like symbol manipulation but also understanding mathematical issues in context. [1].

In a traditional mathematics classroom, the teacher explains the theory and the students solve the exercises. In his work, Ponte (2011) highlights an alternative approach to teaching mathematics when the teacher gives a task and a certain amount of time to the students, after which the students present their solutions and argue in order to find the most suitable solution. In another article, the authors say that inquiry-based approach depends mainly on the teacher rather than on the choice of tasks [2]. Once a task has been chosen, it is important for the teacher to consider how to propose the exploration of the problem in the classroom (Stein, Engle, Smith, Hughes, 2008). Evaluating mathematical explanations in an Inquiry-Based Mathematics Classroom is a very difficult task, taking into account the mathematical component itself and the established norms in the classroom [3].

Inquiry-Based Learning is a form of active learning that involves deep engagement in mathematics and gives students the opportunity to collaborate (Lauren, Kogan, 2014). Students themselves perform a huge part of the decision-making process on the way to solving the problem. In addition, they actively work as a team on mathematical ideas before making decisions. In lessons focused on proofs, students explain their proofs in front of the class [4]. The others peer-review the explanation for its validity, reliability and consistency. Therefore, “class discussion is the collaboration of the entire class” (Dana C. Ernst, Angie Hodge, Stan Yoshinobu, 2017). The successful application of Inquiry-Based Learning depends on the number of students, their background, teachers’ skills and other factors. Menezes, Canavarro, Oliveira (2012) provide in their work with stages of intentional actions of the teachers in an inquiry-based classroom practice, starting from the launching the task to the students and supporting them to work on a task independently, ending with organizing of the task discussion and systematizing math learning. Levenson (2013) characterized the explanation in an inquiry-based mathematics classroom by two components, description of mathematics and justification by referring to the mathematics. According to this, students have to perform effectively, and their thinking and ways of explanations should be mathematically valid and understandable to other classmates [5].

According to the recent studies, the application of authority, sociomathematical norms, and mathematical practices in the classroom affects students’ ability to evaluate and afterwards leads to failure of students to evaluate mathematical explanations based on mathematics (GabarielaPurnamaNingsi, FransiskusNendi, 2021). In the papers of Kadir, Jafar, Jazuli, Ikman (2018) Sociomathematical norms are indicated as a normative behavior of students in the math class, and this is one of the ways students participate in all math activities in the classroom. Cobb and McClain (2001) in their research indicate three sociomathematical norms regarding students’ explanations of their own work and evaluations of others’ explanations [7]. In the work of Fukawa-Connelly (2012), those three sociomathematical norms looked different depending on the class where they are applied. Since sociomathematical norms specialize specifically in the study of mathematics, therefore sociomathematical norms are social norms associated with area of mathematics [8]. In order to understand mathematical concepts, students interact using mathematical reasoning or argumentation (Sulfikawati, Suharto, & Kurniati, 2016).

The abovementioned authority is a teaching authority, expressed in the form of teachers’ control over all learning processes in the classroom, which causes low student activity, resulting in low student ability to evaluate mathematical explanations based on mathematics, and this in turn is the reason of poor mathematics learning outcomes (Sumaryati et al., 2013). When a teacher empowers his students to act as an authority in the classroom, Wagner & Herbel-Eisenmann (2014) called it “sharing authority”, teacher controls the students who can provide with correct explanation of mathematical problems. According to Pramudya et al. (2020), classroom mathematical practice is a teaching practice used in math classrooms that includes all math-learning processes from beginning to end of learning [9].

We can find papers about different sides of student evaluation and explanation in mathematics classroom, but there seems to be not enough reviews in this field. The purpose of the present work is to deliver a review of the scientific output of this topic [10].

In the following Table 1, you can see the range of previous publications concerning the student evaluation and explanation in mathematics classroom and application of Inquiry-Based Learning. The review contains a total of 13189 articles, of which 82 relative articles have been selected.

Table 1. Number of publications from 2002 to 2022.

Keywords/Years	2002-2006	2007-2011	2012-2016	2017-2022	Total
<b>Evaluation of mathematical explanations</b>					
anywhere in the article	-	-	1	1	2
<b>sociomathematical norms</b>					
anywhere in the article	681	1060	1480	1680 (80)	4910
in the title of the article	2	13	18	23 (1)	56
<b>“classroom mathematical practices”</b>					
anywhere in the article	174	271	335	340 (18)	1140
in the title of the article	4	1	1	4	10
<b>“teaching authority”</b>					
anywhere in the article	1320	1710	1910	2140 (80)	7270
in the title of the article *26 papers were irrelevant	1	1	3	3	8*
<b>“inquiry-based mathematics classroom” or “inquiry-based teaching mathematics”</b>					
anywhere in the article	6	21	39	23	89
in the title of the article	-	-	5	1	6
<b>total</b>	<b>2181</b>	<b>3062</b>	<b>3762</b>	<b>4184</b>	<b>13189</b>

Considering the purpose of the study, we propose the following research questions:

RQ1. How have scientific outcomes changed over the past 20 years in student evaluation of mathematical explanations in an Inquiry-Based Mathematics Classroom?

RQ2. What is the distribution by country of the research papers in the field of student evaluation of mathematical explanations?

RQ3. What are the mostly covered research topics in the area of student evaluation of mathematical explanations?

In the following parts of this work, we present the applied research method, the results and their discussion. The final step is to summarize and make conclusions based on the results of the work.

**Research method.** The first step in the literature review starts from searching the research papers in scientific literature databases. This part deals with the selection of data sources and search criteria.

The applied method for this literature survey is a bibliometric review. Common reviews present the relevant features of a scientific work and provides with accurate and essential information. Research articles were searched in Google Scholar database using “evaluation of mathematical explanations”,

“sociomathematical norms”, “teaching authority”, “classroom mathematical practices”, “inquiry-based mathematics classroom” as a keywords(search with quotation marks), excluding citations and patents, date limit 2002-2022. These words were chosen as mandatory words, and they had to be in at least one of the three positions in the selected article: in the title, abstract or keywords.As a result of the review, found more than 13000 scientific papers and 82 relevant ones were selected from them (after selection of related articles by reading abstracts).The final selection of related papers is done by reading the abstracts. Google Scholar is used frequently in other systematic literature reviews. It’s main advantage is open access compared to other databases. The search done in the April 2022 [11 ].

At the end of this stage, to refine the results, all the collected materials were presented in the form of a table.

Table 2. The search criteria.

Step	Search criteria	Results
1	Initial queries, the keywords anywhere in the article	13189
2	Selection of articles, the keywords in the title of the article	108
3	Filtering by language: English	105
4	Final selection of related articles by reading abstracts	82

**Results and discussion.** The first research question regards the scientific outcomes in the last 20 years in student evaluation of mathematical explanations in an Inquiry-Based Mathematics Classroom [12]. The chart 1 presents the number of publications by year, by initial queries, where the keywords occur anywhere in the article. As seen on the chart, the number of published articles is increasing year by year, and indicators for the last 5 years are twice those of the first period of the study (2002-2006).



Chart 1. Number of published research papers from 2002 to 2022.

The second research question concerns the country distribution of the relevant research papers in the field of student evaluation of mathematical explanations, after a final selection of the related works by reading abstracts. The

chart 2 shows the countries in the dataset [ 13 ]. The absolute leader is the United States of America with 29 papers, followed by Turkey and Indonesia with 10 and 9 works respectively. United Kingdom has 4, followed by other countries such as Canada, Republic of Korea and South Africa, each of them has 3 research works.

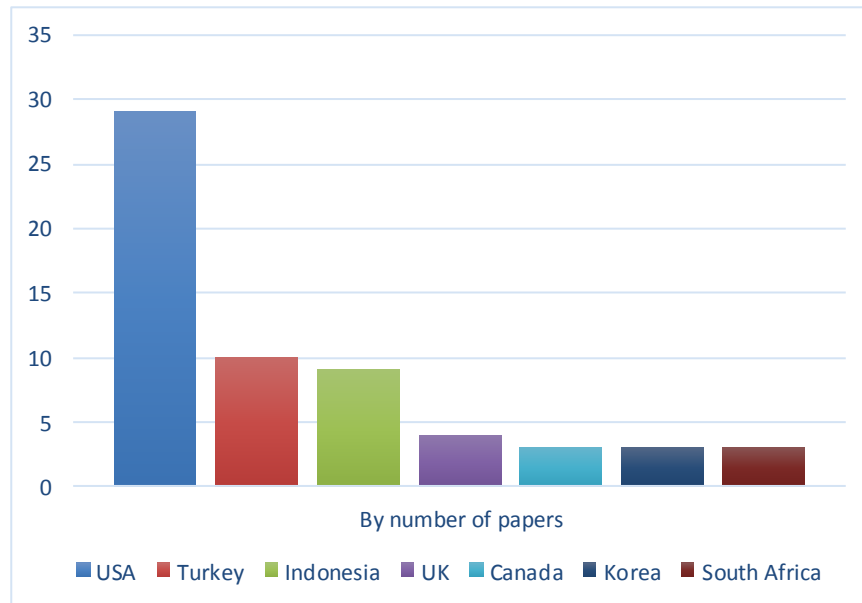


Chart 2. Country distribution of the reviewed research papers.

The last research question concerns the mostly covered research topics in the area of student evaluation of mathematical explanations. The abstracts of the articles were read in order to highlight the most important topics. The chart 3 presents the four main research areas and the number of published papers. The works not included in the main topics were excluded from the chart. The category “sociomathematical norms” has a greatest number of publications, 56 relevant papers were chosen from 4910. On the other hand, the “inquiry-based mathematics classroom” has less research papers, 6 from 89.

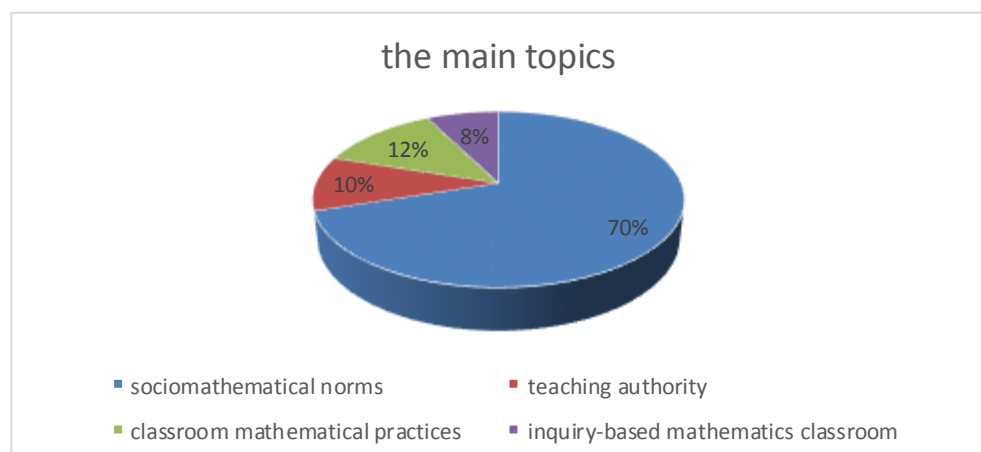


Chart 3. The main topics and publications.

The application of the limited type of bibliometric review is best suited for answering the research questions.

Regarding the first research question, the number of publications in the second decade, period from 2012 to 2022, is greater than in the first decade, period from 2002 to 2012. The constant growth of research works is obvious.

The answer to the second research question shows that the United States of America has made significant scientific contributions in the field of student evaluation of mathematical explanations, followed by Turkey and Indonesia, each accounting for one-third of US papers [ 14 ].

The third research question is dedicated to the most important topics in the area of student evaluation of mathematical explanations. Approximately 70% of all chosen papers belong to the “sociomathematical norms”. Many authors indicate these norms as the main ones that primarily affect students in the process of evaluating mathematics explanations in mathematics classroom. However, only a few authors used qualitative analysis to support the above thesis. This kind of research is limited to the students’ evaluation of mathematical explanations in mathematics classroom, so for the study authors used the case study method (Polit and Beck, 2004, Zainal, 2007) [ 15 ].

Along with the students’ background, motivation, educational and socio-cultural factors, learning strategies, self-related cognitions in mathematics, these factors - sociomathematical norms, teaching authority and classroom mathematical practices - also affect the academic performance of students. For this reason, these factors will be further investigated.

**Conclusions.** In this paper, we have performed a literature review by using the limited type of bibliometric method, and our purpose was to analyze the scientific output in the area of evaluation of mathematical explanations by students in an Inquiry-Based Mathematics Classroom in the past 20 years (2002-2022). Difficulties in evaluating an explanation are one of the big challenges experienced by students. There are several reasons why students are unable to evaluate mathematical explanations: the mathematics teaching and learning activities in schools are teacher-centered, in which the teacher still holds the authority as the sole evaluator in learning activities. Students should be able to explain math problems according to established norms in the classroom, such as a teaching authority, sociomathematical norms, and math class practice [ 16 ].

The research questions regard the number of publications in this field, distribution by countries and the most important topics. 13189 articles were found, of which 82 relative articles were selected. The leaders in the number of research papers – the United States of America, Turkey and Indonesia. The most represented subtopic is sociomathematical norms (70%) [ 17 ].

By using Google Scholar database, we analyzed the research topics and types. The limitation of Google Scholar database – there is no options to limit the searching process by abstracts or keywords. The suggestion for further reviews – the use of additional databases is an advantage, but as a rule, they are on a fee basis.

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**МАТЕМАТИКА САБАҒЫНДА ОҚУШЫНЫҢ МАТЕМАТИКАЛЫҚ  
ТҮСІНДІРМЕЛЕРДІ БАҒАЛАУЫ ТУРАЛЫ ЗЕРТТЕУЛЕРГЕ ШОЛУ**

**Аңдатпа.** Мақалада математика сабағында оқушылардың математикалық түсіндірмелерді бағалау бағытындағы алдыңғы жарияланымдар қарастырылады. Көптеген студенттер өз жұмыстарын түсіндіруге тартыншақтайды, сыныптастары берген түсініктемелерге күмәнданады. Сонымен қатар, студенттер есептердің шешімдерін сирек түсіндіреді және математикалық түсініктемелерді сирек

бағалайды. Оқушының білімі, оқу және әлеуметтік-мәдени факторлармен қатар, бұл факторлар — әлеуметтік-математикалық нормалар, мұғалімнің беделі және сыныпта қалыптасқан математикалық тәжірибе — студенттердің оқу үлгеріміне де әсер етеді.

Мақаланың мақсаты – соңғы 20 жылдағы (2002-2022 жж.) математика сабағында математикалық түсініктемелерді оқушылардың бағалауы саласындағы зерттеушілердің ғылыми өнімділігі мен әсерін анықтау. Бұл жұмыста шолудың нәтижелері мен талдауы берілген. Зерттеу сұрақтарына жауап беру үшін библиометриялық әдістің шектеулі түрін пайдалану ең қолайлы болды.

Математикалық түсіндірмелерді оқушылардың бағалауы тақырыбы бойынша жарияланымдарды іздеу үшін Google Scholar деректер базасы пайдаланылды.

**Тірек сөздер:** математикалық түсіндірмелерді бағалау, әлеуметтік және математикалық нормалар, мұғалім беделі, сыныпта қалыптасқан математикалық тәжірибе.

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#### **ОБЗОР ИССЛЕДОВАНИЙ НА ТЕМУ ОЦЕНКИ СТУДЕНТАМИ МАТЕМАТИЧЕСКИХ ОБЪЯСНЕНИЙ В КЛАССЕ МАТЕМАТИКИ**

**Аннотация.** В этой статье рассматриваются предыдущие публикации в области оценки математических объяснений учащимися в классе математики. Многие ученики не решаются объяснить свою работу и сомневаются в объяснениях своих одноклассников. Кроме того, студенты редко объясняют решение задачи и редко оценивают математические объяснения. Наряду с фоном учащихся, образовательными и социокультурными факторами эти факторы - социально-математические нормы, преподавательский авторитет и математическая практика в классе - также влияют на академическую успеваемость учащихся.

Цель статьи - определить научную продуктивность и влияние исследователей в области оценки студентами математических объяснений в классе математики, основанном на запросах, за последние 20 лет (2002-2022). В данной работе представлены результаты и анализ обзора. Применение ограниченного типа библиометрического метода лучше всего подходит для ответа на вопросы исследования.

База данных Google Scholar использовалась для поиска публикаций на тему оценки студентами математических объяснений.

**Ключевые слова:** оценка математических объяснений, социально-математические нормы, преподавательский авторитет, математические практики в классе.